

We Probably Live On An Inflating Brane-World ^{*}

Ishwaree P. Neupane[†]

Department of Physics and Astronomy, University of Canterbury
Private Bag 4800, 8041 Christchurch, New Zealand

31 March 2011

Abstract

Brane-world models where observers are trapped within the thickness of a 3-brane offer novel perspectives on gravitation and cosmology. In this essay, I would argue that the problem of a late epoch acceleration of the universe is well explained in the framework of a 4-dimensional de Sitter universe embedded in a 5-dimensional de Sitter spacetime. While a 5D anti de Sitter space background is important for studying conformal field theories – for its role in the AdS/CFT correspondence – the existence of a 5-dimensional de Sitter space is crucial for finding an effective 4D Newton constant that remains finite and a normalizable zero-mode graviton wave function.

^{*}(Slightly expanded version of) essay written for the Gravity Research Foundation 2011 essay contest.

[†]E-mail: ishwaree.neupane@canterbury.ac.nz

Cosmology is a young science – one which attempts to reconstruct and explain the entire history of the universe from nearly 13.8 billions of years ago. This cosmic history should include a brief period of rapid exponential expansion of the early universe, known as ‘inflation’, which provides the most credible explanation to how causal physics in a very early epoch produced the large scale structures that we observe today. However, looking back so far in time (via observations) is extremely difficult and an added difficulty is that many of the theoretical pillars of physics upon which the models of inflation and late epoch cosmic acceleration (attributed to dark energy) rest have only been proposed within the last 3 decades or so. That hasn’t given cosmologists (and astrophysicists) much time to fully flesh out and comprehend the situation. There are two distinct possibilities: either we’re missing new physics and hence going to find a fatal flaw in our prevailing view of the universe’s present composition, or that we encounter interesting discoveries and surprises in cosmology and particle physics experiments waiting ahead of us.

It is true that our understanding of the physical universe has deepened profoundly in the last few decades through thoughts, observations and experiments. It is also true that the concurrent universe still has a number of cosmological mysteries, but the one that has most puzzled physicists is the smallness of the present value of gravitational vacuum energy (or ‘dark energy’), i.e. $\Lambda_{\text{obs}} \lesssim (10^{-3} \text{ eV})^4$ and its eminent effect on an accelerated expansion of the universe at a late epoch [1]. There is no shortage of ideas for how to construct a model which is capable to produce a universe with de Sitter-type expansion. There is a large gamut of gravitational theories (see, e.g. [2, 3]) that are capable to explain an accelerated expansion of the universe with certain modification of Einstein’s theory of general relativity. In this essay, I would argue that the problem of a late epoch acceleration as well as the smallness of the cosmological constant may be well explained within the framework of a de Sitter 3-brane embedded in a 5D de Sitter spacetime (dS₅).

The issue of a late epoch cosmic acceleration (or the dark energy problem) is perhaps not about difficulties of finding a particular cosmological model which could mimic as the Lambda-CDM cosmology, described by Einstein gravity with a cosmological constant and minimally coupled to both the luminous (baryonic) and non-luminous (cold dark) matter. The challenge is to come up with a fully consistent theory in four-dimensions that explains the origin of cosmic acceleration, while providing insights into some other major problems in physics, including the mass hierarchy problem in particle physics. More generally, we should be able to explain why the constants of the standard model of cosmology, including the dark energy and dark matter densities, have the values they do.

The paradigm that the physical universe is a brane-like 4-dimensional hypersurface embedded in a higher dimensional space is fascinating [4, 5]. This very idea is inspired by a theoretical framework of scientific thoughts and also by fundamental theories of gravity, particles and fields, such as string and M theory, which provide novel approaches for unifying Einstein’s general relativity with quantum field theories. Obtaining 4-dimensional de Sitter solutions from dynamical compactifications of higher dimensional theories has been an important issue in string cosmology. Since an epoch of cosmic acceleration plays an important role in modern cosmological models, it would be very interesting to know

whether this effect can be explained within the framework of string theory and/or modified theories of gravity, such as, brane-world gravity in higher dimensions [6, 7].

An attractive feature of 5D brane-world models is that the standard 4-dimensional gravity, not very different from Einstein's theory, is realized as the zero-mode solution of a 5D graviton wave equation. In the simplest RS brane-world proposal [5], the background spacetime geometry is a 5D Anti de Sitter space (AdS_5), which is warped. The all 4 spatial dimensions are noncompact, but one of them behaves differently for particles and fields which are confined within the brane's thickness. There also exists a massless graviton as the zero-mode solution, which reproduces the standard Newtonian gravity on the 3-brane [8, 9, 10, 11]. The Kaluza-Klein modes arising as the effect of graviton fluctuations in the bulk spacetime give rise to corrections to the Newton's force law [12, 13, 14].

The RS brane-world model [5] corresponds to a static universe for which the Hubble expansion parameter is zero. This original setup is not suitable for describing a realistic cosmology for which space and time are not independent (or the 3-brane is dynamical). Furthermore, AdS_5 is perhaps *not* the most preferred background geometry for a universe to experience a de Sitter-type expansion at a late epoch. The main reason being that the standard 4D gravity, which may be viewed as the zero-mode solution of a 5D graviton wave equation, is not necessarily normalizable if the background geometry is AdS_5 .

To quantify this, one may consider a 5-dimensional metric of the following form

$$ds^2 = e^{2A(z)} (\gamma_{\mu\nu} dx^\mu dx^\nu + dz^2), \quad (1)$$

where $e^{2A(z)}$ is the warp factor, and look for a class of solutions for which the 4D line-element is the standard Friedmann-Lemaître-Robertson-Walker (FLRW) metric,

$$ds_4^2 \equiv \gamma_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right], \quad (2)$$

where k is the 3D curvature constant with the dimension of inverse length squared. The relevant 5D Einstein-Hilbert (gravitational) action is

$$S_{\text{grav}} = M_{(5)}^3 \int d^5x \sqrt{-g} (R - 2\Lambda_5) + \int \sqrt{-\gamma} (-\tau), \quad (3)$$

where $M_{(5)}$ is the 5D Planck mass and τ is the 3-brane tension. If the size of the physical three dimensions does not change with time or $ds_4^2 = \eta_{\mu\nu} dx^\mu dx^\nu \equiv -dt^2 + dx^2 + dy^2 + dz^2$, then the zero-mode gravity solution is normalizable only when (i) the bulk cosmological constant is negative, i.e. $\Lambda_5 \equiv -6/\ell^2$, where ℓ is the radius of curvature of the 5D bulk spacetime, and (ii) the 3-brane tension satisfies the relation $\tau = 12M_{(5)}^3/\ell$.

In the limit $\Lambda_5 \rightarrow 0$, the 5D bulk spacetime is spatially flat and gravity is *not* localized. In this case the solution is trivial, i.e. $A(z) = 0$, $a(t) = 1$. However, when $\Lambda_5 = 6/\ell^2 > 0$ and $a(t) \propto e^{Ht}$, the solution becomes nontrivial, which is given by

$$e^{A(z)} = \frac{\ell H}{\cosh Hz}, \quad \tau = \frac{6M_{(5)}^3}{\ell} \sinh H z_c, \quad (4)$$

where $z_c (> 0)$ is a constant. Upon the dimensional reduction from 5D to 4D, we find that the 4D effective Planck mass M_{Pl} is related to the 5D Planck mass $M_{(5)}$ via

$$M_{\text{Pl}}^2 = \frac{\pi \ell^3 H^2}{2} M_{(5)}^3. \quad (5)$$

Note that unlike in the RS static brane-world models, for which $\ell_{\text{AdS}} \lesssim 0.01$ mm, the de Sitter curvature radius ℓ can be significantly larger; if we want $M_{(5)} > 10^{-15} M_{\text{Pl}}$ and $H = H_{\text{obs}} \sim 10^{-61} M_{\text{Pl}}$, then $\ell \lesssim 10^{55} \ell_{\text{Pl}} \sim 10^{22}$ cm. The 4D Planck mass is well defined even in the limit $H \rightarrow 0$ since when the thickness of the brane becomes infinitely large (or $H^{-1} \rightarrow \infty$), ℓ can take a naturally large value. In such a case the background 5D geometry is only modestly warped so as to balance the effect of a small positive curvature associated with 4D cosmological constant on the de Sitter 3-brane and $\Lambda_4 = 6H^2$ [15].

Let us now consider the linear perturbations of the 5D metric $\delta^{(5)}g_{AB} = h_{AB}$. For $\Lambda_5 > 0$, we find that the transverse-traceless tensor modes $\delta g_{ij} = h_{ij}(x^\mu, z) \equiv \delta_i^\mu \delta_j^\nu h_{\mu\nu}(x^\mu, z) = \sum \alpha_m(t) e^{-3A/2} \psi_m e^{ik \cdot x} \hat{e}_{ij}$ satisfy the following Schrödinger-type wave equation

$$\frac{d^2 \psi_m}{dz^2} - V(z) \psi_m = -m^2 \psi_m, \quad (6)$$

where

$$V(z) = \frac{9H^2}{4} - \frac{15H^2}{4 \cosh^2(Hz)} - 6H \tanh(Hz) \delta(z - z_c). \quad (7)$$

The brane is located at $z = z_c$. The zero-mode solution ($m^2 = 0$) is given by

$$\psi_0(z) = \frac{b_0}{(\cosh(Hz))^{3/2}}, \quad (8)$$

which is clearly normalizable since

$$\int_{-\infty}^{\infty} |\psi_0(z)|^2 dz = \frac{\pi b_0^2}{2H}.$$

There is one more bound state solution, i.e.,

$$\psi_1(z) \propto \frac{\sqrt{\cosh^2(Hz) - 1}}{(\cosh(Hz))^{3/2}}, \quad (9)$$

which is obtained by taking $m^2 = 2H^2$. This solution is also normalizable. However, only the zero-mode solution ($m^2 = 0$) is localized on the de Sitter 3-brane [14].

Around the brane's position at $z \equiv z_c$, satisfying $H z \ll 1$, the solution looks like

$$\psi(z) = c_1 P_\mu + c_2 Q_\mu, \quad (10)$$

where

$$P_\mu = 1 - \frac{3+2\zeta}{4}(Hz)^2 + \dots, \quad Q_\mu = Hz - \frac{3+2\zeta}{12}(Hz)^3 + \dots, \quad (11)$$

and $\zeta \equiv m^2/H^2 \geq 9/4$. This solution corresponds to a situation that the thickness of the brane is much larger than the size of the fifth dimension, $H^{-1} \gg z$. In the large zH limit,

$$\psi_{Hz \rightarrow \infty} = c_1 e^{i\mu z H} + c_2 e^{-i\mu z H}, \quad (12)$$

where $\mu \equiv \sqrt{\zeta - \frac{9}{4}}$. The masses of Kaluza-Klein modes are quantized in units of H . With $c_1 = 0$, and away from the brane's position, satisfying $z \gg H^{-1}$, all heavy modes with $\mu > 0$ become oscillating plane waves, which represent the de-localized KK modes.

To estimate the correction to Newton's force law generated by a discrete tower of Kaluza-Klein modes, one may go to the thin brane limit, i.e. $H^{-1} \rightarrow 0$, but keeping the ratio z_c/H^{-1} finite. One would also assume that the matter fields in the 4-dimensional theory is smeared over the width of the brane and the brane thickness is smaller compared with the bulk curvature, $H^{-1} < \ell$, so $H\ell > 1$. Under these approximations, the gravitational potential between two point-like sources of masses M_1 and M_2 located on the brane is modified via exchange of gravitons living in 5 dimensions as

$$U(r) \simeq \frac{G_4 M_1 M_2}{r} \left(1 + \frac{2\alpha}{\pi} \frac{M_{\text{Pl}}^2}{M_{(5)}^3 r} \sum_i e^{-m_i r} \right), \quad (13)$$

where $m_i \geq \sqrt{2}H$, r is the distance between the two pointlike sources, and α is a constant of order unity. The correction to the gravitational potential due to the massive KK states may dominate for $r < H^{-1} < \ell$, leading to a 5-dimensional behavior which is not Newtonian. However, since the corrections to the gravitational potential are suppressed by a factor of $\sum_i e^{-m_i r}$, with $m_i \gtrsim \text{TeV} \sim 10^{-15} \text{ cm}$, the deviation from Newton's inverse law may not show up unless we probe a sufficiently small distance scale, $r \lesssim 10^{-12} \text{ cm}$.

For $\Lambda_5 < 0$, the analysis closely follows the one given above, but the opposite sign of Λ_5 modifies the above results. The Schrödinger-type potential $V(z)$ in (6) is given by

$$V(z) = \frac{9H^2}{4} + \frac{15H^2}{4 \sinh^2(Hz)} - 6H \coth(Hz) \delta(z - z_c). \quad (14)$$

The zero-mode solution ($m^2 = 0$) is now given by

$$\psi_0(z) = \frac{c_0}{(\sinh(Hz))^{3/2}}. \quad (15)$$

In this case it is necessary to have \mathbb{Z}_2 -symmetry, or allow a cutoff scale, $0 < z_c \leq z$; otherwise the zero-mode solution is non-normalizable.

A similar analysis can be carried out by introducing scalar fields [16]. Here we study a 5D gravity action minimally coupled with a canonical scalar field ϕ with potential $U(\phi)$:

$$S_{\text{grav}} = \frac{1}{2} \int d^5x \sqrt{-g} \left[\frac{R}{\kappa_5^2} - g^{AB} (\partial_A \phi) (\partial_B \phi) - 2U(\phi) \right], \quad (16)$$

where $\kappa_5^2 \equiv 1/M_{(5)}^3$. For simplicity, we begin with the $k = 0$ (spatially flat) case. It is natural to assume that ϕ depends only on z . The 5D field equations now take the form

$$\phi'^2 = \frac{3}{\kappa_5^2} (A'^2 - A'' - H^2), \quad U(\phi) = \frac{3e^{-2A}}{2\kappa_5^2} (3H^2 - 3A'^2 - A''), \quad (17)$$

where $' \equiv d/dz$. These equations admit a series of solutions with different choice of $\phi(z)$ or $A(z)$. For illustration, we look for a particular solution of the form

$$A(z) = \beta - \lambda \ln \cosh \left(\frac{Hz}{\lambda} \right), \quad (18)$$

where β and λ are some constants. This leads to a standard domain-wall-type solution

$$\kappa_5 \phi = \phi_0 - \phi_1 \arcsin \tanh \left(\frac{Hz}{\lambda} \right), \quad (19)$$

where ϕ_i are dimensionless constants, and $\phi_1 = \sqrt{3\lambda(1-\lambda)}$. Note that ϕ behaves as a canonical scalar field when $0 < \lambda < 1$. The scalar potential $U(\phi)$ is now given by

$$U(\phi) = \frac{3(1+3\lambda)H^2}{2\lambda\kappa_5^2} \left[\cos^2 \left(\frac{\phi_0 - \kappa_5 \phi}{\phi_1} \right) \right]^{(1-\lambda)}. \quad (20)$$

This potential is of Mexican-hat type (as long as $0 < \lambda < 1$) and the maximum occurs at $z = 0$. In fact, the solutions (18)-(20) are available also when the 3D spatial curvature k is nonzero, i.e. $a(t) = \frac{a_0}{2} e^{Ht} + \frac{k}{2H^2 a_0} e^{-Ht}$. For $k \neq 0$, the choice $H = 0$ is *not* physical.

To obtain an effective 4D theory, we shall consider the dimensionally reduced action

$$S_{\text{eff}} = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g_4} (R_4 - 2\Lambda_4), \quad (21)$$

where

$$M_{\text{Pl}}^2 = e^{3\beta} \frac{M_{(5)}^3 \lambda}{H} \int \frac{d\varphi}{(\cosh \varphi)^{3\lambda}}, \quad \Lambda_4 = e^{3\beta} \frac{M_{(5)}^3 H}{M_{\text{Pl}}^2} \int \frac{(6\lambda \cosh^2 \varphi - 3\lambda - 1)}{(\cosh \varphi)^{2+3\lambda}} d\varphi \quad (22)$$

and $\varphi \equiv (Hz/\lambda)$. Note that the integral

$$I_\lambda \equiv \int_{-\infty}^{\infty} \frac{d\varphi}{(\cosh \varphi)^{3\lambda}} = \frac{\sqrt{\pi} \Gamma[3\lambda/2]}{\Gamma[(3\lambda+1)/2]} \quad (23)$$

is finite with $\lambda > 0$. This shows that the 4D effective Planck mass M_{Pl} is also finite. That is, as in the model without a scalar field, the effective 4D Newton's constant is finite despite having a noncompact extra dimension. One may extend the above discussion for a general class of D -dimensional warped supergravity models, including 10-dimensional

supergravity coupled with scalar fields and form field strengths. It is possible to obtain a 4-dimensional de Sitter universe as the exact solution of classical supergravities in general $D = 4 + n$ dimensions [17]. It is also possible to stabilise the scale (or size) of the internal space by suitably choosing fluxes (i.e. the gauge field strengths wrapped around the internal manifold), the curvature term associated with the internal space and the bulk cosmological constant, leading to a dynamical mechanism of warped compactification.

I conclude with a few remarks.

One of the most exciting developments in cosmology is the suggestion that the warping of extra dimensions plays a key role in explaining mass hierarchy and localization of gravity on a de Sitter 3-brane, embedded in a higher dimensional spacetime. dS_5 space allows a foliation by a flat space but that is a spacelike hypersurface. There is no way to cut dS_5 by Minkowski spacetime. That is, in a cosmological setting, a 4D Minkowski spacetime (or a flat 3-brane) is not a solution to 5D Einstein equations, if the bulk spacetime geometry is de Sitter. This is in contrast to the results in Randall-Sundrum brane-world models in AdS_5 spacetimes. But this is anyway not a problem since the universe has probably never gone through a phase of being close to a static universe or a flat 3-brane.

While a 5D anti de Sitter space background is important for studying conformal field theories – for its role in the AdS/CFT correspondence – the existence of a 5-dimensional de Sitter space is crucial for finding an effective 4D Newton constant that remains finite and a normalizable zero-mode graviton wave function. The 5-dimensional model discussed above is found to admit both an effective 4-dimensional Newton constant that remains finite and a normalizable graviton wave function.

In this essay, we tried to explore a much desired connection between 5D de Sitter gravity and FRW-type cosmologies built upon the idea of the existence of an extra noncompact dimension and a warped background geometry in five dimensions. The important question that we seek to address is whether or not a canonical theory of brane-world gravity admits a lower dimensional description in which gravity emerges from it in what would now be called a consistent warped compactification to 4D de Sitter spacetime.

It may well be that the final theory of quantum gravity requires a highly symmetric (or even supersymmetric) background geometry in 1+9 spacetime dimensions, such as $AdS_5 \times S^5$ - a product space of a 5-dimensional Anti de Sitter space and a five-sphere. But such geometry wouldn't remain as a stable background once the universe starts to expand or undergoes an inflationary de Sitter phase. Nevertheless, it's possible to obtain a realistic cosmology in four-dimensions by allowing spacetime backgrounds such as $dS_5 \times S^5$ and $dS_5 \times T^{1,1}$. This outcome possibly implies that a spontaneous breaking of supersymmetry corresponds to a tunnelling of the universe from an AdS_5 to a dS_5 state.

Acknowledgement

This work was supported by the Marsden fund of the Royal Society of New Zealand.

References

- [1] A. G. Riess *et al.* Astron. J. **116** (1998) 1009; S. Perlmutter *et al.*, Astrophys. J. **517** (1999) 565.
- [2] E. J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. **D15**, 1753-1936 (2006).
- [3] I. P. Neupane, arXiv:0711.3234 [hep-th].
- [4] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136; M. Visser, Phys. Lett. **B159** (1985) 22.
- [5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690.
- [6] I. P. Neupane, Class. Quant. Grav. **26** (2009) 195008.
I. P. Neupane, Class. Quant. Grav. **27** (2010) 045011.
- [7] I. P. Neupane, Phys. Lett. B **683** (2010) 88.
- [8] R. Maartens, Living Rev. Rel. **7** (2004) 7.
- [9] I. H. Brevik, K. Ghoroku, S. D. Odintsov and M. Yahiro, Phys. Rev. D **66** (2002) 064016.
- [10] A. Kehagias and K. Tamvakis, Class. Quant. Grav. **19** (2002) L185.
- [11] U. Gen and M. Sasaki, Prog. Theor. Phys. **105** (2001) 591.
- [12] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84** (2000) 2778.
- [13] K. Ghoroku, A. Nakamura and M. Yahiro, Phys. Lett. B **571** (2003) 223.
- [14] I. P. Neupane, Phys. Rev. **D83** (2011) 086004.
- [15] I. P. Neupane, Int. J. Mod. Phys. D **19** (2010) 2281.
- [16] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62** (2000) 046008.
- [17] I.P. Neupane, Nucl. Phys. **B847** (2011) 549